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Developing Creative Problem Solvers

Eric L. Mann
Hope College, mann@hope.edu

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Developing Creative Problem Solvers

Math In Action

Grand Valley State University

Feb 25, 2017

Eric L. Mann, PhD
mann@hope.edu

PDF file of this presentation is available at https://goo.gl/Z2FDNU

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Overheard in math class: "Oh God why is this happening to me? I go to church!"

"We have known for some years now…that most children's mathematical journeys are in vain because they never arrive anywhere, and what is perhaps worse is that they do not even enjoy the journey."

Whitcombe, A. (1988), Mathematics creativity, imagination, beauty. *Mathematics in School, 17,* (2), 13-15

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The U.S. does not need fast procedure executors anymore. We need people who are confident with mathematics, who can develop mathematical models and predictions, and who can justify, reason, communicate, and problem solve. **We need a broad and diverse range of people who are powerful mathematical thinkers and who have not been held back by stereotypical thinking and teaching.**

Jo Boaler, Professor, Mathematics Education, Stanford University

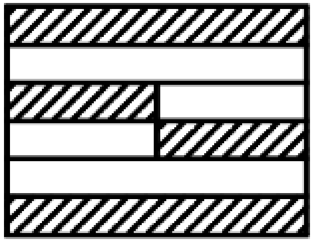
The Stereotypes That Distort How Americans Teach and Learn Math

[The Atlantic Online (12 Nov 2013)]

The 3Rs: Recite, Replicate, Regurgitate

Fractions Grade 6: Draw a rectangle with a 3 cm base and a 4 cm height. Shade 3/6th thereof.

Solution of a pupil:



While going through the classroom, that pupil asked me [the teacher] whether or not his solution was correct. *I was forced* to *admit that it was*. That is *what you get when you don't tell the pupils exactly what to do* . . . The teacher now reproaches himself for *not having prevented this solution.* He is obviously influenced by an insufficient understanding of what is mathematics, by the image of school as an institution for stuffing of brains . . . (p. 88)

Köhler, H. (1997). Acting artist-like in the classroom. *ZDM Mathematical Education, 29*(3), 88–93.

Constant emphasis on sequential rules and algorithms may prevent the development of creativity, problem solving skills, and spatial ability.

Pehkonen, E. (1997). The state-of-art in mathematical creativity. ZDM Mathematics Education, 29 (3), 63-37

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2007 bi-annual conference: International Community of Teachers of Mathematical Modeling and Applications

Co-hosts: Indiana University, Purdue University's INSPIRE, USAFA

"The Air Force Academy is a good example...where they came to us with a problem and they said **'our cadets come in here; they're smart kids. They come out knowing more, and they get worse on absolutely every scale of being good problem solvers, of being creative.'** They know more and can function less, in a way. And they're very worried, because **the person that they need** in the military just like other things, for the future, **isn't somebody who just follows rules. They need to understand those and be able to create their own flow of them.** So having them engaged helps us get on the forefront of things."

Dr, Richard Lesh, Professor of Learning Sciences, Cognitive Science, and Mathematics Education, Indiana University

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Negative Numbers

- Initially used as a computational tool
  - 200 BC: The Chinese number rod system
  - 620 AD: Brahmagupta (India) fortunes and debts
- **A controversial topic for centuries**
  - **300 AD: Diophantus**: The solution to 4 = 4x + 20 would be **"absurd"**
  - **Descartes (1637): False, fictitious numbers**
  - **Carnot (1803)** to obtain an isolate negative quantity, it would be necessary to cut off an effective quantity from zero, to remove something from nothing: **impossible operation.**
  - **Busset (1843)**
    - **attributed the "failure of the teaching of mathematics in France to the admission of negative quantities."**
    - compelled to declare that **such mental aberrations could prevent gifted minds from studying mathematics**

For more on the history of Negative Number start at NRICH, The University of Cambridge http://nrich.maths.org/5961 and Anne Boyé's essay, *Some Elements of the History of Negative Numbers.*

Teaching and Learning of Mathematics 1894*

The method of teaching should be throughout objective, and such as to call into exercise the pupil's mental activity. The text-books should be subordinate to the living teacher.

* National Educational Association (1894). *Report of the Committee of Ten on secondary school studies with the reports of the conferences arranged by the committee.* New York: American Book Company. (pg. 105).

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Teaching and Learning of Mathematics 1894*

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* National Educational Association (1894). *Report of the Committee of Ten on secondary school studies with the reports of the conferences arranged by the committee.* New York: American Book Company. (pg. 105).

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Teaching and Learning of Mathematics 2012

Common Core State Standards for Mathematical Practice (NGA & CCSSO, 2010)

1. Make sense of problems and preserve in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critiques the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

Using the Common Core State Standards for Mathematics with Gifted and Advanced Learners (Johnsen & Sheffield, 2012)

9. Solve problems in novel ways and pose new mathematical questions of interest to investigate.

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Teaching and Learning of Mathematics 2012

21st Century Skills Math Map	
Mathematical Association of America National Council of Teachers of Mathematics The Partnership for 21st Century Skills	
Learning and Innovation Skills	**Creativity and Innovation:** Students use a wide range of techniques to create new and worthwhile ideas, elaborate, refine, analyze and evaluate their own ideas in order to improve and maximize creative efforts, and demonstrate originality and inventiveness.
	Critical Thinking and Problem Solving: Students reason effectively, use systems thinking and understand how parts of a whole interact …make judgments, decisions and solve problems in both conventional and innovative ways.
	Communication and Collaboration: Students know how to articulate thoughts and idea effectively…listen effectively to decipher meaning…(communicate) for a wide range of purposes.

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The Power of Mathematical Conversation

- Project Challenge (Chapin & O'Conner, Boston University)
- 4 year intervention focusing on discourse-based teaching in the **lowest performing school district in Massachusetts**
  - 400 4th graders, 70% low-income, 60% ELL
  - Teachers trained to use a variety of talk moves to encourage student to explain their reasoning and build on one another's thinking
  - **After 2 years the proportion of student showing a "high probability of giftedness in mathematics" as measure on the Test of Mathematical Abilities (TOMA) rose from 4% to 41%**
  - **After 3 years 82% of Project Challenge students scored "Advanced" or "Proficient" on the state assessment** (state average proficiency was 38%)

Resnick, L. B., Michaels, S. & O'Conner, M.C. (2010). How (well-structured) talk builds the mind R. J. Sternberg & Priess D.D. (Eds). *Innovations in Educational Psychology*, (163-194). New York: Springer.

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Dan Meyer's: Math Class Needs A Makeover

Five symptoms that you're developing math reasoning skills wrong. Your students…

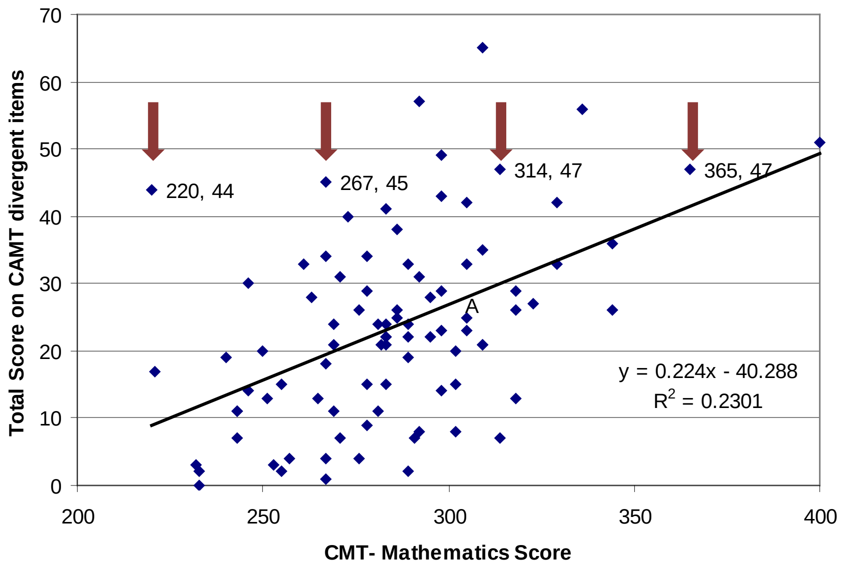
1. Lack initiative
2. Lack perseverance
3. Lack retention
4. Are adverse to word problems
5. Are eager for a formula

Two and Half Men approach to learning math:

- Teaching in small, "sitcom sized problems that wrap up in 22 minutes, 3 commercial breaks and a laugh track" resulting in **Impatient Problem Solvers**

Creativity in School Mathematics?

- An individual's knowledge base is the fundamental source of their creative thought. (Feldhusen and Westby, 2003)
- Students with equal mathematical achievement may have significant differences in performance on measures of mathematical creativity (Haylock, 1997)



Post Hoc Regression Analysis found that Achievement scores were a significant predictor of performance for those who scored below the mean creativity score but not above.

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Creative Ability in Mathematics

The ability to

1. Formulate mathematical hypotheses
2. Determine patterns
3. Break from established mind sets to obtain solutions in a mathematical situation
4. Sense what is missing and ask questions
5. Consider and evaluate unusual mathematical ideas, to think through the consequences from a mathematical situation
6. Split general mathematical problems into specific sub-problems

Balka, D. S. (1974). Creative ability in mathematics. Arithmetic Teacher, 21, 633–636.

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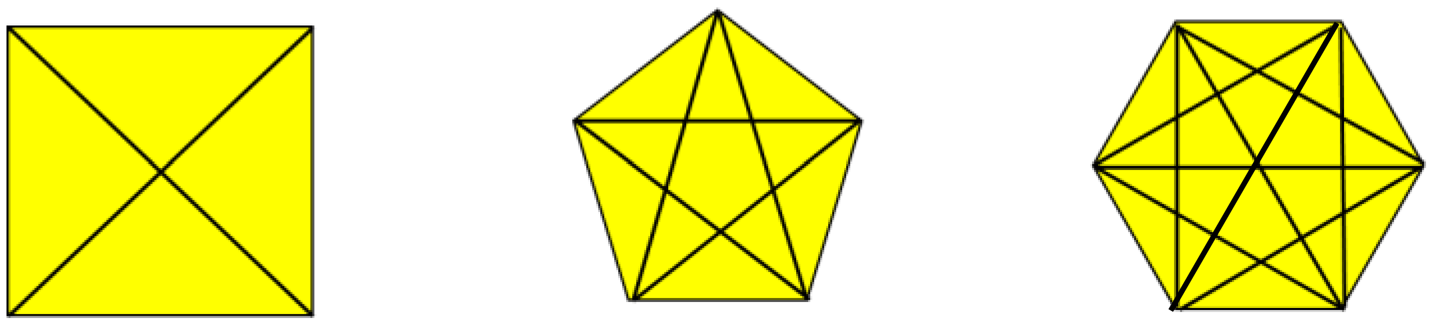
What is your hypotheses?



Is there a connection between the number of sides of a polygon and the number of diagonals you can make?

Problem is loosely structured – a research question

Cause: change in number of sides
Effect: change in number of diagonals

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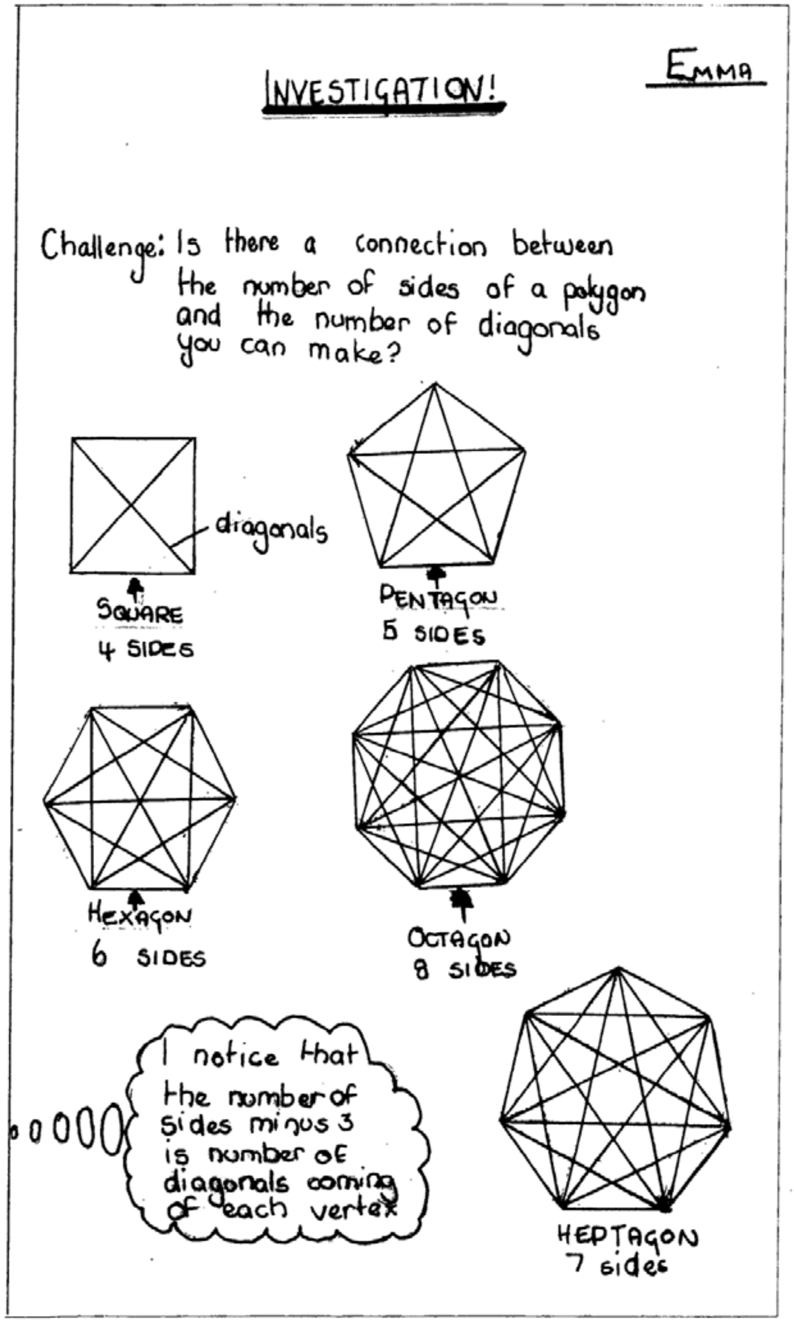

Emma's Work (Casey, 2011)

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Emma's Work

Emma

Name Of Shape	Num Of Sides	Num Of Diagonals
Quadrilateral	4	2
Pentagon	5	5
Hexagon	6	9
Heptagon	7	14
Octagon	8	20

I noticed that the number of diagonals of a 4,5,6,7,8 sided shape increase in a pattern (2+3=5+ 4=9,+5=14,+6=20)

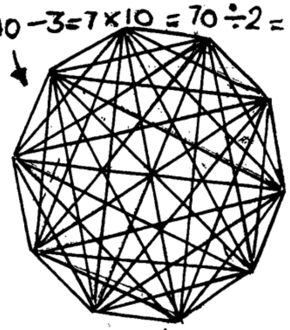
THE RULE

I think its sides − 3 × $\frac{\text{sides}}{2}$ = diagonal

eg: Pentagon 5−3=2 ×5 =10 ÷2 = amount of diagonals: 5

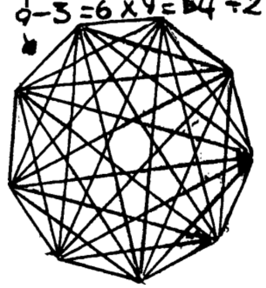
This works with any shape

10−3=7×10=70÷2=35



DECAGON
10 sides

9−3=6×9=54÷2=27



NONAGON
9 sides

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Emma's Work

There is a notation issue (S-3 × S/2 = d) will not generate the data chart Emma created.

The equation she used was (S-3) × S/2 = d

A teachable moment rather than a wrong answer.

I can make a formula from what I know. This formula is:

FORMULA

$(S-3 \times \frac{S}{2} = d)$

KEY

S= sides
d= diagonals

Now I can predict any polygon's diagonals

CHART

Sides	Number of diagonals
20	170
21	189
22	209
23	230
40	740
80	3,080
160	12,560
320	50, 720

What I notice about My Chart

The first 4 numbers (20, 21, 22, 23) still go up in a pattern. 170 + 19 = 189 + 20 = 209 + 21 = 230. Strangely, in my pattern, double the number (sides) is not double the diagonals

"Strangely in my pattern double the number (sides) is not double the diagonals."

An opportunity to extend the problem with a student generated inquiry.

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Emma's Voice

I tried to find the connection between the number of sides in each shape and the number of diagonals which were drawn. So I drew out a chart to help me look at the relationship between the numbers. I wondered what relation the numbers had to each other. When I looked at the number of diagonals in each shape I noticed that the difference between them increases by 1 each time. When I looked at the shape drawings again, I noticed that the numbers of diagonals coming from each vertex was three less than the number of sides in the shape eg. pentagon. 5 – 3 = 2 diagonals from each vertex, hexagon 6 – 3 = 3 etc. This gave me a rule for the number of diagonals coming from each vertex in any particular shape, but I needed to find the number of diagonals in each shape.

I tried to do what I thought of before. I tried to multiply the 2 × 5 (ie. the number of diagonals by the number of sides of the pentagon because there were five sides in the shape) and I noticed the answer was 10. I then noticed that 10 is twice the number of sides of a pentagon, and if I divided it by 2 it would give me the total number of diagonals, which it did. I then tried to do the same with the other shapes:

octagon (8 – 3 = 5) × 8 then divide the total by 2 = 20

nonagon (9 – 3 = 6) × 9 then divide the total by 2 = 27

This works with all the other shapes, it also allows me to predict the number of diagonals in any shape (Landers, 1999)

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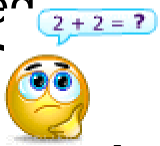
Emma’s Voice

I tried to **find the connection** between the number of sides in each shape and the number of diagonals which were drawn. So I drew out a chart to help me look at the relations… between the numbers. I wondered what relation the numbers had … other. When I looked at the number of diagonals in each shape … between them increases by 1 each tim… drawings again, I noticed that the number… each vertex was three less than the number of sides in the shape eg. pentagon. 5 – 3 = 2 diagonals from each vertex, hexagon 6 – 3 = 3 etc.)

Not search for a formula!



Emma's Voice

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Her first strategy

Emma’s Voice

I tried to find the connection between the number of sides in each shape and the number of diagonals which were drawn. So I drew out a chart to help me look at the relationship between the numbers. I **wondered** what relation the numbers had to each other. When I looked at the numbe of diagonals in each shape I **noticed** that the difference between them incr ses by 1 each time. When I looked at the shape drawings again, I **noticed** the numbers of diagonals coming from each vertex was three less than th ber of sides in the shape eg. pentagon. 5 – 3 = 2 diagonals from ea 3 = 3 etc. This gave me a rule for the number of d vertex in any particular shape, but I needed to each shape.

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Wondering and noticing, not just punching numbers into a calculator in hopes that an answer will appear

The Voice of Emma’s Teacher

. . . from a starting point which involved all children in the class, provides evidence of what **children can achieve through being trained to think**. Previously, Emma would not have even attempted this investigation in spite of her mathematical talent. More importantly, I may not have given this kind of 'difficult' task to the class!

The class was **asked to investigate** if there was a connection between the number of sides of a polygon and the number of diagonals you can draw. I introduced this activity to the whole class. **Every child participated in the initial discussion** on the names and properties of polygons, in defining what diagonals are including demonstrations using people. Logo experiences were sought for further understanding of these concepts. From this common starting point, **children were encouraged to follow lines of enquiry** which matched their capability. What Emma has demonstrated here is a way of working she has acquired in the last few months.

This way of working systematically and constantly refining her thoughts and processes has also enabled Emma to identify similar patterns in other problems and investigations.

Water Jug Problem

- You have three jugs A, B, and C
- The problem is to find the best way of measuring out a given quantity of water, using just three jugs.
- You are told how much each jug holds.
- There are no marks on the jugs so the only way to make an accurate measurement is to fill a jug to the brim.

An Example

- Measure out 55 units if Jug A holds 10 units, Jug B holds 63 units and Jug C hold 2 units.
- Solution:
  - Fill B (63 units)
  - Pour water from B to fill A (53 units left)
  - Fill C (2 units)
  - Add to B (55 units)
  - B – A + C = 63 – 10 + 2 = 55 units

Your Turn

	Measure out	Jug A holds	Jug B holds	Jug C holds	Solution
1	52 units	10	64	1	
2	14 units	100	124	5	
3	3 units	10	17	2	
4	100 units	21	127	3	
5	20 units	23	49	3	
6	5 units	50	65	5	

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A Common Solution

	Measure out	Jug A holds	Jug B holds	Jug C holds	Solution
1	52 units	10	64	1	B-A-2C
2	14 units	100	124	5	B-A-2C
3	3 units	10	17	2	B-A-2C
4	100 units	21	127	3	B-A-2C
5	20 units	23	49	3	B-A-2C
6	5 units	50	65	5	B-A-2C

I see a pattern!

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A more elegant one

	Measure out	Jug A holds	Jug B holds	Jug C holds	Solution
1	52 units	10	64	1	B-A-2C
2	14 units	100	124	5	B-A-2C
3	3 units	10	17	2	B-A-2C
4	100 units	21	127	3	B-A-2C
5	20 units	23	49	3	A-C
6	5 units	50	65	5	C

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Luchnis 1936 Version

	Measure out	Jug A	Jug B	Jug C	Solution	
1	20 units	29	3	-	A - 3B	
2	100 units	21	127	3	B - A - 2C	
3	99 units	14	163	25	B - A - 2C	
4	5 units	18	43	10	B - A - 2C	
5	21 units	9	42	6	B - A - 2C	
6	31 units	20	59	4	B - A - 2C	
Don't be blind!						
7	20 units	23	49	3	B - A - 2C	A - C
8	18 units	15	39	3	B - A - 2C	A + C
9	25 units	28	76	3		A - C
10	22 units	18	48	4	B - A - 2C	A + C
11	6 units	14	36	8	B - A - 2C	A - C

The Einstellung Effect

- The Einstellung effect: set successful procedure applied consistently even when it is less than efficient or inappropriate
- The jug problem was given to 250, 11-12 year olds
  - 70% used the same procedure on all 6 problems
  - 11% used a different procedure on 1 problem
  - 11% used a different procedure on 2 problems
  - 8% couldn't do the problems

(Haylock, 1984)

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Incubation Time and Einstellung Effect

Given four (4) separate pieces of chain that are each three (3) links in length.

- It costs 2¢ to open a link and 3¢ to close a link.
- All links are closed at the beginning of the problem.
- Your goal is to join all 12 links of chain into a circle at a cost of no more than 15¢.

Control	Group 1	Group 2
• Worked for ½ hour • 55% solved the problem	• Worked for ½ hour then ½ hour break • 64% solved the problem	• Worked for ½ hour then 4 hour break • 85% solved the problem

- Students often choose particular method to solving a problem
- If not appropriate, they may be stuck with the method
- Taking a break may allow other methods a chance or students to gain a deeper understanding of the problem.

The cheap-necklace problem experiment (Silveira, 1971)

Problem Formation

- *The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill. To raise new questions, new possibilities, to regard old problems from a new angle, requires creative imagination, and marks real advances in science.*

  (Einstein, The Evolution of Physics)
- Problem formulation - a neglected but essential means of mathematical instruction (Kilpatrick, 1987)
- "…performance in mathematics courses, up to the college and even early graduate levels, often does not effectively predict who will succeed as a mathematician**. The prediction failure occurs due to the fact that in math, as in most other fields, one can get away with good analytical but weak creative thinking until one reaches the highest levels of mathematics**" (Sternberg, 1996)

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Hollenstein’s experiment (cited in Köhler, 1997)

- One group of children worked on a mathematics exercise presented in the traditional method.
  - three-segment lesson: correction of the previous day’s homework, teacher presentation of new material and student practice
  - problems in the experiment were constructed so that a single correct answer existed
- The second group was tasked to develop and answer problems that could be solved using calculations. They …
  - … created and answered more questions.
  - … calculated more accurately.
  - … arrived at more correct results.

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Pyramid of Pennies by Dan Meyer

More on the Penny Project

- Marcelo Bezos founded the Penny Pyramid Project to raise funds for cancer research.
  - His web site
  - You Tube time lapse construction video at a bit slower pace

- Dan Meyer's Web links
  - The lesson materials for the Pyramid of Pennies task
  - Using the lesson at the Centre for Mathematical Sciences, University of Cambridge in 2013 and a follow-up blog post
  - A collection of Three Act Tasks

- Modeling with Mathematics – a discussion of Three-Act Tasks with additional resources from The Teaching Channel

DISCOVERING THE ART OF MATHEMATICS
MATHEMATICAL INQUIRY IN THE LIBERAL ARTS

- The **Discovering the Art of Mathematics**[1] project provides a wealth of resources to support college faculty in teaching Mathematics for Liberal Arts, including a library of 11 inquiry-based learning books, professional development opportunities, and extensive teacher resources.

[1] This project is based upon work currently supported by the National Science Foundation under NSF1225915 and previously supported by NSF0836943 and a gift from Mr. Harry Lucas.

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DISCOVERING THE ART OF MATHEMATICS
MATHEMATICAL INQUIRY IN THE LIBERAL ARTS

Notes to the Explorer

… this book cannot be read in the traditional sense. For this book is really a guide. It is a map. It is a route of trail markers along a path through part of the world of mathematics. This book provides you, our explorer, our heroine or hero, with a unique opportunity to explore this path - **to take a surprising, exciting, and beautiful journey along a meandering path through a mathematical continent** … And this is a vast continent, not just one fixed, singular locale. “Surprising?” Yes, surprising**. You will be surprised to be doing real mathematics. You will not be following rules or algorithms, nor will you be parroting what you have been dutifully shown in class or by the text.** Unlike most mathematics textbooks, **this book is not a transcribed lecture followed by dozens of exercises that closely mimic illustrative examples.** Rather, after a brief introduction to the chapter, the majority of each chapter is made up of Investigations

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https://www.artofmathematics.org/

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- Student Comments
  - Instead of falling asleep listening to lectures **I was able to solve problems and make conjectures.**
  - This course is a breath of fresh air. It helps me understand why math professors enjoy math so much. **I see the fun in math now and how beautiful it can be.**
  - This class taught me **how to think independently** about not only math but other subjects and everyday problem solving.
  - **The fact that it was never easy to find an answer to the problem made me want to find it so much more.**
  - …Math is no longer a student-engagement subject. Students are not given the time or encouraged to experiment with a math problem and find patterns for solving it.…[In this course] **I was encouraged and guided to engage in making the discoveries and understandings for myself.**

Doing What Mathematicians Do

- **Mathematics** when it is finished, complete, all done, then it consists of proofs. But, when it is discovered, it **always starts with a guess**. . .

  George Pólya (1966)

- Mathematics - this may surprise you or shock you some - is never deductive in its creation. **The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions**. He arranges and rearranges his ideas, and he becomes convinced of their truth long before he can write down a logical proof. **The conviction is not likely to come early - it usually comes after many attempts, many failures, many discouragements, many false starts.**

  Paul Halmos (1968)

Questions / Discussion

Contrary to popular belief, mathematics is passionate subject. Mathematics are driven by creative passions that are difficult to describe but no less forceful than those that compel a musician to compose or an artist to paint.

Theoni Pappas, Mathematical Scandals, 1997

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