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The Value of Problem Posing in Developing Creatively Gifted Mathematicians

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General description

- Much has been written about creatively gifted mathematics students. However, little attention has been invested in how to develop prospectively creative mathematics students.
- Generally, conceptions of creativity or ways to identify creatively gifted mathematicians, are presented.
- In this session, presenters discuss the value of problem posing as a means to develop creativity in mathematics.
- To conclude, you will be provided with a mathematical concept and asked to pose relevant mathematical problems. Problems posed will be shared with peers to conclude the presentation.

The Art of Problem Posing

(Brown and Walter, 2005, 3rd Ed.)

- Math Anxiety
  - "Often our formal education system suppresses the relationship between asking questions … and the coming up with answers" (p .3)
  - Fear of being "stuck" or not doing things "the right way"
  - Implications of the "the right way" syndrome and it's impact on creative and critical thinking
- Why Problem Posing?
  - "Problem posing has the potential to create a totally new orientation toward the issue of who is in charge and what has to be learned." (p.5)
  - Helps students see topics in a shaper light; acquire deeper understanding
  - Encourage creation of new ideas
  - Shift emphasis from the learning of mathematics as a "spectator sport" to a "participant sport"
  - "Impossible to solve a new problem without first reconstrucing the task by posing new problems" (p.2)
  - "After we have supposedly solved a problem we do not fully understand the significance of what we have done, unless we begin to generate and try to analyze a completely new set of problems"

A laconic overview of the literature

- Lavy & Shriki (2007): Through a problem posing activity, prospective teachers developed their ability to examine
  - definition and attributes of mathematical objects
  - connections among mathematical objects
  - validity of an argument." (p. 129).
- PTs tended to focus on commonly posed problems, being afraid of their (in)ability to prove their findings.
- PTs overemphasis on providing formal proofs prevented the development of inquiry activities.

Literature review

- Leung (1997): Leung focused on the relationship between creative thinking and problem posing in mathematical domains.
- Leung found that fluency is general in verbal creativity and problem posing, but flexibility is specific in problem posing. Further, problem posing abilities (in mathematics) exist in elementary students.
- Shriki (2013): Through problem posing, Shriki suggested a model for assessing students' creativity with an emphasis on four measurable creative outcomes, including: fluency, flexibility, originality and organization, and a total score of creativity.

Assessing the Development of Students' Creativity (Shriki, 2013

- Fluency: Number of new problems posed
- Flexibility: Number of different categories of problems
- Originality: For a problem to be considered orginal it needs to be posed by a subset of the refence group (Skriki set the threshold at ≤ 33%
- Organization: Problems formualted as a generalization
- Creativity "Scoring": Based on reference group with the highest score in each criterion set at 100

Literature review

- Silver (1997): Silver argued that creativity is not necessarily restricted to gifted students and therefore a curriculum rich in opportunities to solve and pose problems can help students develop many creative approaches to mathematics.
- Silver (1994): Silver suggested that the only types of problems students solve in mathematics are ones in textbooks ☹, not ones that peers generate. He further stated that to help students make sense of mathematics, they need to be able to solve problems posed by peers.

Mathematical Dispositions

Common Core Standards for Mathematical Practice
(NGA & CCSSO, 2010)

1. Make sense of problems and preserve in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critiques the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

Using the Common Core State Standards for Mathematics with Gifted and Advanced Learners (Johnsen & Sheffield, 2012)

9. Solve problems in novel ways and pose new mathematical questions of interest to investigate.

Problem: Is there a connection between the number of sides of a polygon and the number of diagonals you can make?

Problem is loosely structured – a research question

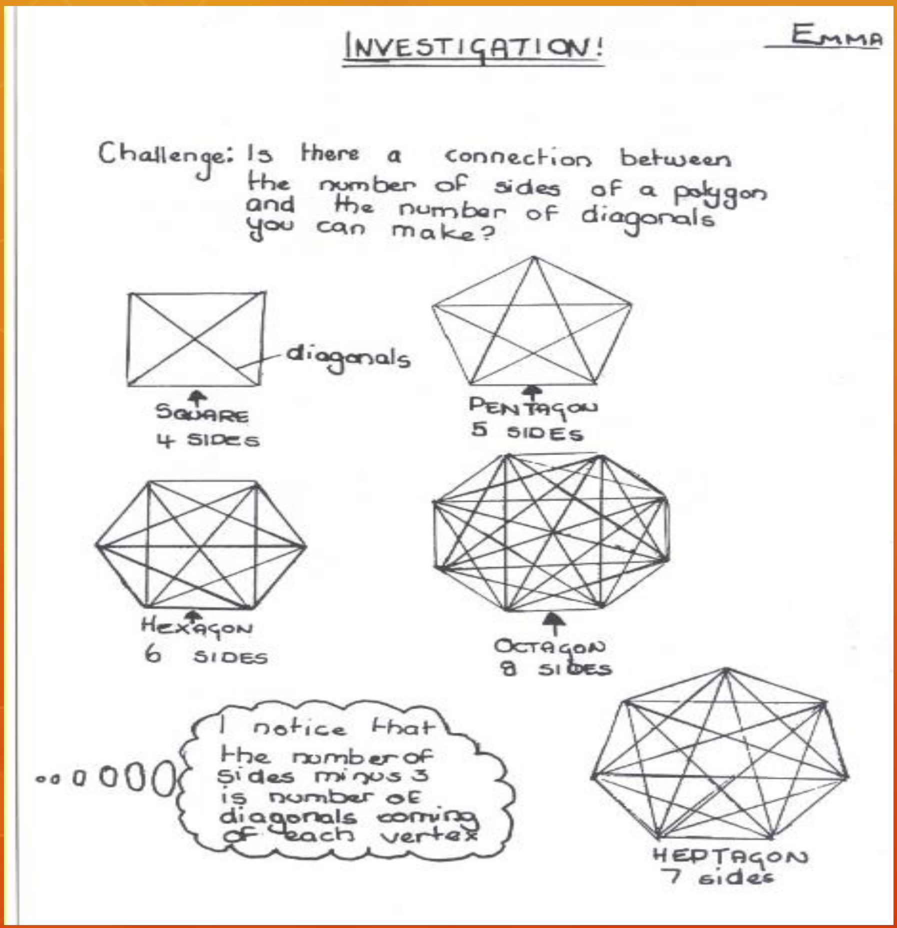
Cause: change in number of sides
Effect: change in number of diagonals

Emma’s Voice

- I tried to find the connection between the number of sides in each shape and the number of diagonals which were drawn.
- I wondered what relation the numbers had to each other.
- I noticed that
  - When I looked at the number of diagonals in each shape the difference between them increases by 1 each time.
  - When I looked at the shape drawings again, the numbers of diagonals coming from each vertex was three less than the number of sides in the.
- This gave me a rule for the number of diagonals allows me to predict the number of diagonals in any shape (Landers, 1999)

Emma's Work s 9 years old)

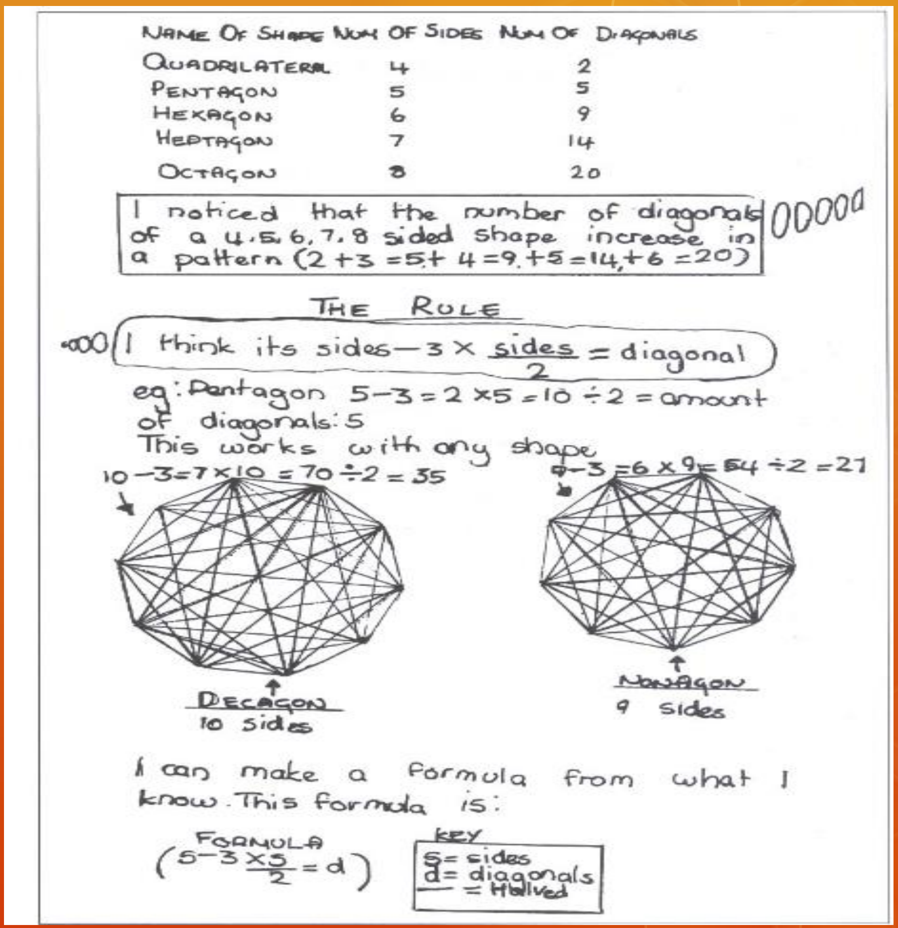
(Koshy, 2001, p. 43-44]



INVESTIGATION!

EMMA

Challenge: Is there a connection between the number of sides of a polygon and the number of diagonals you can make?



NAME OF SHAPE	NUM OF SIDES	NUM OF DIAGONALS
QUADRILATERAL	4	2
PENTAGON	5	5
HEXAGON	6	9
HEPTAGON	7	14
OCTAGON	8	20

I noticed that the number of diagonals of a 4.5.6.7.8 sided shape increase in a pattern (2 +3 =5+ 4 =9 +5 =14,+6 =20)

THE RULE

I think its sides—3 × sides/2 = diagonal

eg: Pentagon 5−3=2 ×5 =10 ÷2 = amount of diagonals: 5

This works with any shape

I can make a formula from what I know. This formula is:

FORMULA

(s−3 × s/2 = d)

KEY

s = sides
d = diagonals
— = Halved

The Voice of Emma's Teacher

- . . . from a starting point which involved all children in the class, provides evidence of what children can achieve through being trained to think. Previously, Emma would not have even attempted this investigation in spite of her mathematical talent.
- Every child participated in the initial discussion on the names and properties of polygons, in defining what diagonals are
- From this common starting point, children were encouraged to follow lines of enquiry which matched their capability.
- This way of working systematically and constantly refining her thoughts and processes has also enabled Emma to identify similar patterns in other problems and investigations.

(Landers, 1999)

What If Not – the WIN strategy
(Brown & Walter, 2005)

- The game of NIM – Problem Posing with Elementary Students (Song, Yim, Shin & Lee, 2007)
- Two Phases: (1) Accepting (2) Challenging
  - Accepting: Twenty units of yellow coloured tubes are connected with one unit of black tube. Two students take turns to take from one to three cubes. The student who takes the last cube is the winner
  - Challenging: Creating new rules of the game or posing new problems
    - Place a black cube at the center with 7 yellow cubes on its left side, and 13 red cubes on its right side. Two students will take turns to take at least one and up to three cube of the same color. The student who takes the black cube will be the loser.

Fibonnacci Series and What If Not

(Brown & Walter, 2005, pgs 66 – 74)

Statring with 1 and 1 as the first terms add any two adjacent terms and the sum will yeild the next one 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 …

Level 1: Accepting (Understanding)

- Attributes
  - We start with two given numbers
  - The first two given numbers are the same
  - The same number is 1
  - If we do something to any two successive numbers we get the next number
  - The something we do is an operation
  - The operation is addition
  - The sequence of ratios of successive terms approaches $(\sqrt{5} - 1) \div 2 \cong .618$
  - The difference between any two adjacent terms generates another Fibonnacci Sequence
  - The square of any term differs by one from the product of two adjacent terms.

Fibonnacci Series and What If Not

(Brown & Walter, 2005, pgs 66 – 74)

- Level 2 Challenging Attributes
  - Suppose 10 and 7 are the first two numbers
- Level 3 Asking Questions
  - What limit, if any, does the ratio of succeeding terms in the new series approach?
  - How does the square of any term compare with product of the two adjacent terms in the new series?
- Level 4 Analyzing Questions
  - Seeking answers – finding new questions

Share your responses

- What problems did you create?
- Next, we are going to share some problems with you. Some of these, were designed by us and some are problems that we have accumulated over a number of years.

Task for you

- We promised that you would have the opportunity to problem pose today. Here are several concepts. Select one, or more, and try to pose a problem that may be of interest to students.
  - Algebra: Pattern thinking
  - Geometry: Pythagorean Theorem (DOES ANYONE HAVE A BETTER PROBLEM THAN THE ONE I USED???)
  - Measurement: Assigning a quantitative value to an attribute
  - Number & Operation: Sum consecutive numbers
  - Probability: Randomness
  - Statistics: Measures of central tendency

Algebra: Pattern Thinking

Locker Problem

One hundred students have lined up in a very long hallway with 100 closed lockers. One by one the students run through the hall and perform the following ritual.

- The first student opens every locker.
- The second student goes to every second locker and closes it.
- The third student goes to every third locker and changes its state; that is, if it's open, the student closes it, if it's closed, the student opens it.
- In a similar manner the fourth, fifth, sixth...student changes the state of their respective lockers.

After all 100 students have passed down the hallway, which lockers are open and which are closed?

Geometry: Pythagorean Theorem

Ladder problem

- A maintenance worker needs to transport a ladder to the agriculture department in a large rural high school. Given the dimensions listed below, what is the largest ladder that can be transported?

- Note: the hallway turns at 90 degrees and is 15 feet in width.

Measurement: Assigning a value

The Big Foot Problem

- Early this morning, the police discovered that, sometime late last night, some nice people rebuilt the old brick drinking fountain in the park. The mayor would like to thank the people who did it. However, nobody saw who it was. All the police could find were lots of footprints. One distinctive, very large footprint was found and is shown below. The police feel this is the person they want to find first because whoever made the footprint seems to be very big. To find this person and his friends, it would help if we could figure out how big he/she really is.

- Your job is to make a “HOW TO” TOOLKIT, a step by step procedure, the police can use to figure out how big people are by looking at their footprints. Your toolkit should work for footprints like the one that is shown here, but it also should work for other footprints.
- Source: https://engineering.purdue.edu/ENE/Research/SGMM/CASESTUDIESKIDSWEB/bigfoot.htm

Number and operations: Sum consecutive numbers

Adding problem

- Add the numbers 1 through 96 and provide the last digit in the final number you attain. Does a mathematical model exist that might enable you to arrive at the answer with a number other than 96 (e.g., 123, 247, 81)?

Probability: Randomness

The iPod® Shuffle

- Looking at the 25 random play lists, determine whether true randomness exists. Write a letter to the Apple Corporation explaining how you came to your conclusion.
- Note: With this problem, you would be provided with 25 random play lists. Problem can be accessed at: http://serc.carleton.edu/sp/library/mea/examples/example6.html

Statistics: Measures of central tendency

On-time arrival

- In the table that follows, you will find information for arrival times for five airline flights for a month. The arrival times are for flights originating at O'Hare Airport (Chicago, ORD) and arriving in Mexico City International Airport (AICM). Rank the five airlines in terms of most likely to be on time to least likely to be on time. As you rank the airlines, document your process carefully so that it can be shared with peers.
- Note: With this problem, you would be provided with on time arrival data from five major (fictitious airlines). The problem can be found at: https://engineering.purdue.edu/ENE/Research/SGMM/CASESTUDIESKIDSWEB/ontimearrival.htm

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